UNPUBLISHED PRELIMINARY DATA

New York University
Courant Institute of Mathematical Sciences
4 Washington Place
New York 3, New York

March 5, 1965

# STATUS REPORT

National Aeronautics and Space Administration Research Grant No. NsG-412 to New York University

Ref.: SC NsG-412 Project Director: J. J. Stoker

Title: Non-Linear Elasticity

Research in the field of this grant has been continued along very similar lines as in the previous report. The principal workers were F. John, E. Reiss, C. Sensenig, J. J. Stoker, and several students. Their contributions are described below.

F. John completed the first phase of a study of Thin Elastic Shells. The problem here was to derive interior two-dimensional equations for suitable quantities describing the location and stresses of the shell from the "exact" non-linear 3-dimensional equations. This can be done by purely formal expansions which would lead to no error estimate in the approximations; but the aim here was rather to derive equations with error estimate. The quality of the resulting approximation depends, of course, on various circumstances about which assumptions have to be made. A priori estimates for the shell equations derived here were given in terms of the thickness of the shell, its curvature, the distance from the edge, and the maximum strain of the shell. Later phases of the project, it is hoped, will become more realistic. Instead of comparing the differential equations one would like to compare their solutions; instead of assuming information on the maximum stress one should get along with the prescribed boundary tractions, or at least with the total potential energy (essentially the $L_2$-norm of the stress).

In the course of the investigation on thin shells, described above, a priori estimates were given for the derivatives of the solutions of the 3-dimensional shell equations. These estimates bear some obvious affinity to the type of estimate found by the use of St. Venant's principle. This suggestion has been pursued in a Ph.D. thesis by Joseph Roseman. Drawing on various methods developed in the theory of elliptic partial differential equations by Lax, Nirenberg and others and with some feeling for the "physical" or "geometrical" situation at hand, Roseman succeeded in deriving rigorously St. Venant's principle for long two-dimensional "bodies", showing exponential decay with distance of all but the resultant and moment of the tractions. Proceeding to the more difficult 3-dimensional case, it happened that the main obstacle had just been removed by the results of Toupin of IBM, and it remained for Roseman to sharpen Toupin's $L_2$-result into a pointwise statement.

Another aspect of elasticity theory is currently being studied by John. This is the problem of possible deformation of solids of a given shape for prescribed maximum strain of the deformation, studied purely geometrically and quite divorced from any considerations of dynamics or material composition, and hence also from any differential equations, apart from the differential inequalities describing the maximum strain. There must be purely geometric reasons why it is harder to make an impression on a solid sphere than on a thin long bar, or why a thin column buckles more easily than a thick one. The present study under the general name of Quasi-Isometric Mappings sets out to investigate this question. Though technically performed in Hilbert-spaces the aim of this investigation is purely 3-dimensional. It incorporates some results published earlier in less complete form, with proofs now no more dependent on the use of coordinate systems, and adds various new results. While these are not yet the best, they already present estimates with the right orders of magnitude, e.g. indicating on purely geometrical grounds the order of thickness-length ratio of a slender column for the strains at which it is likely to buckle. The hope is that perhaps a good deal of elasticity, as far as these orders of magnitude of phenomena are concerned, can be formulated without discussing the solution of partial differential equations at all; roughly speaking one would proceed along the lines of a pure dimensional theory.

E. L. Reiss has continued his work on non-linear buckling of cylindrical shells. The existence of an intermediate buckling load, first conjectured by Friedrichs for spherical shells, was established for several loading situations. Rigorous upper and lower bounds for this load are being obtained.

Most of C. Sensenig's work has been a continuation, both analytic and numerical, on his shell theory in which rounds are deformed into smooth curves with certain properties. In particular, he has been calculating the deformations of a circular plate using this theory. Analytically, he has been estimating the displacement derivatives, the stresses and the stress derivatives in terms of displacements. The ultimate objective is to relate his own theory to Fritz John's asymtotic methods as applied to the case of thin shells.

J. J. Stoker has continued reworking his notes on non-linear elasticity. He has also been directing the work of J. Wolkowisky and R. Dickey. Mr. Wolkowisky is investigating the behavior of a non-linear plate sitting on a spring and loaded at the edge. The idea is to find the buckling modes at the edge. This is a simple model for the crust of the earth and for the formation of mountain ranges on the rim of the ocean. Mr. Dickey has completed a thesis on the dynamic deformation of axially symmetric elastic surfaces. The main concern has been with the displacements of soap films, but work has also been done on rubber-like and classic materials.

## BIBLIOGRAPHY


F. John, "Quasi-Isometric Mappings", Seminari dell'Istituto Nazionale di Alta Matematica 1962-63, (Roma, 1964), pp. 462-473.

"Remarks on the Non-Linear Theory of Elasticity", Seminari dell'Istituto Nazionale di Alta Matematica, 1962-63, (Roma, 1964), pp. 474-482.

"Quasi-Isometric Mappings in Hilbert Space", New York University, Courant Institute of Mathematical Sciences, IMM-NYU 336, January, 1965, p. 65.

"Estimates for the Derivatives of the Stresses in a Thin Shell and Interior Shell Equations", Comm. Pure Appl. Math., 18(1965), (to appear).

"Estimates for the Accuracy of the Non-Linear Shell Equations", Proceedings of the Symposium on Applications of Non-Linear Partial Differential Equations in Mathematical Physics, New York, April 21-3, 1964, (to appear).

C. Sensenig, "Non-Linear Theory for the Deformation of Pre-Stressed Circular Plates and Rings", New York University, Courant Institute of Mathematical Sciences, IMM-NYU 326, June, 1964.